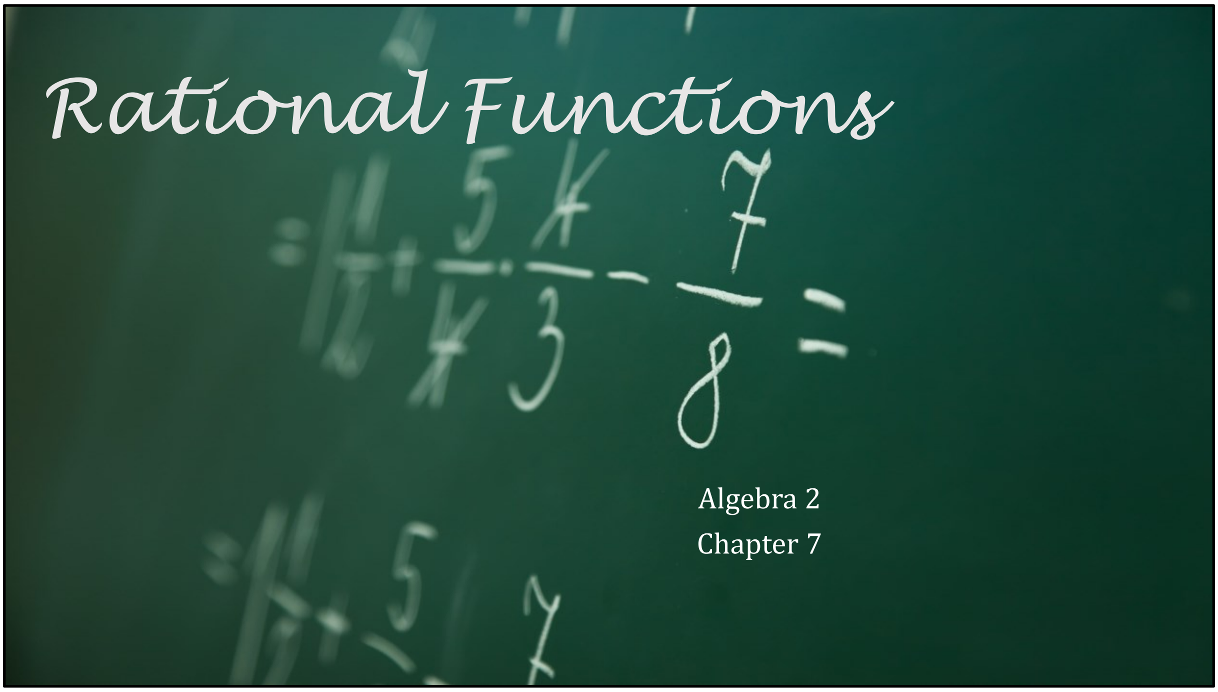
Rational Functions
Algebra 2
Chapter 7

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Algebra 2*
  - *By Larson, R., Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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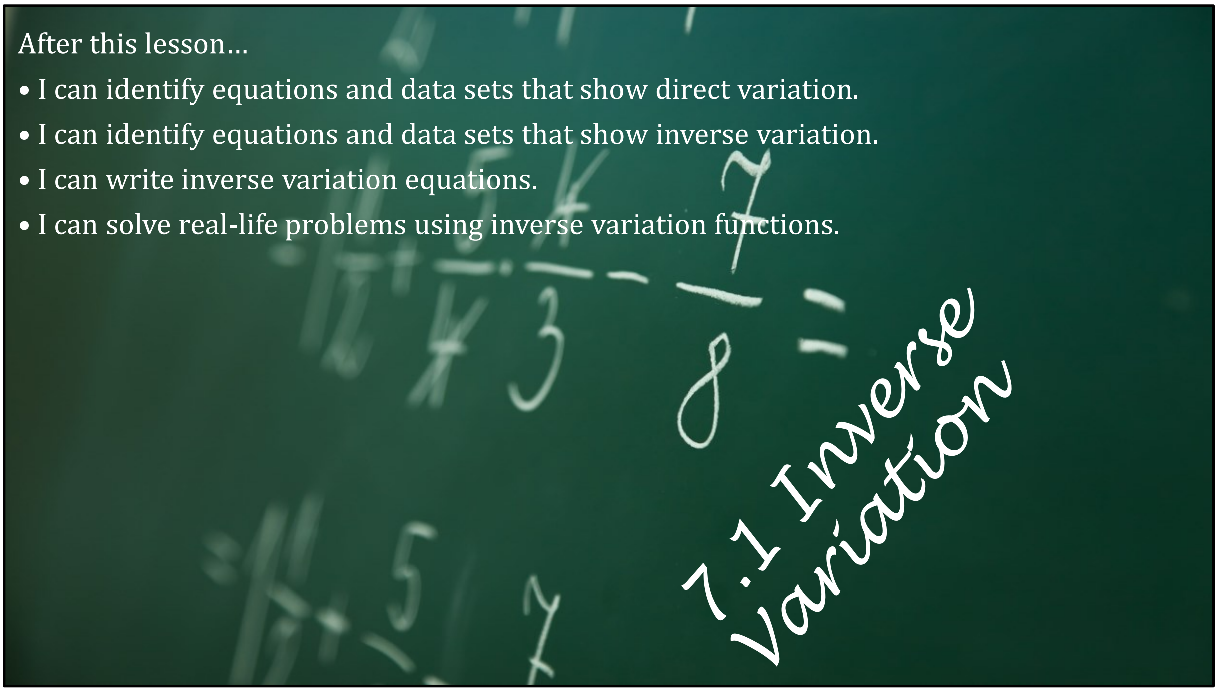

After this lesson…
• I can identify equations and data sets that show direct variation.
• I can identify equations and data sets that show inverse variation.
• I can write inverse variation equations.
• I can solve real-life problems using inverse variation functions.
7.1 Inverse Variation

7.1 Inverse Variation

- Direct Variation: $y = ax$
  - $x\uparrow, y\uparrow$
- Inverse Variation: $y = \frac{a}{x}$
  - $x\uparrow, y\downarrow$

a is the constant of variation

7.1 Inverse Variation

- What type of variation is each of the following?
  - $xy = 48$
  - $2y = x$
  - $y = 2x + 3$
  - Try 359#1
    $y = \frac{2}{x}$

y = 48 / x → inverse

y = ½ x → direct

+3 means neither

359#1: inverse

7.1 Inverse Variation

- Checking data for variation
  - Look at the *y*-values
    - If *y* increases as *x* increases, check direct variation
    - If *y* decreases as *x* increases, check inverse variation
  - Plug each of the data points in one of the variation equations to find *a*
  - If the *a* stays the same, the data has that type of variation
- What type of variation?

x	2	4	8
y	8	4	2

- Try 359#9

y decreases as *x* increases, check inverse variation

$$y = \frac{a}{x} \rightarrow xy = a$$

(2, 8) → *xy* = 2(8) = 16
(4, 4) → *xy* = 4(4) = 16
(8, 2) → *xy* = 8(2) = 16

359#9: direct

7.1 Inverse Variation

- Solving Variations
  - Write the equation in order stated.
    - "Varies" means "$= a$"
  - Plug in x and y to find a
  - Plug in a and the other value and solve
- 359#15: y varies inversely as x. When $x = -3$, $y = 8$. Write an equation relating x and y. Then find y when $x = 3$.
- Try 359#13
- y varies inversely as x. When $x = 5$, $y = -4$. Write an equation relating x and y. Then find y when $x = 3$.

$$y = \frac{k}{x} \rightarrow 8 = \frac{k}{-3} \rightarrow -24 = k$$

$$y = -\frac{24}{x}$$

$$y = \frac{-24}{3} \rightarrow y = -8$$

359#13: $y = -\frac{20}{x}$; $y = -\frac{20}{3}$

7.1 Inverse Variation

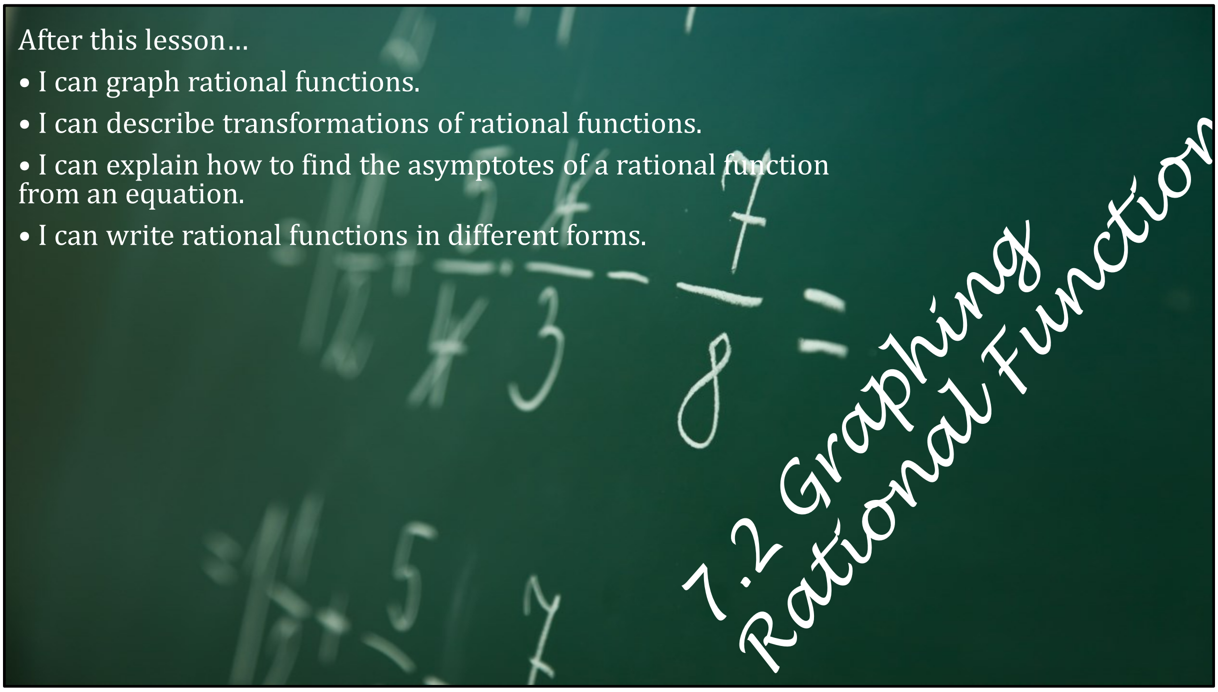
- The time t (in hours) that it takes a group of roofers to roof a house varies inversely with the number n of roofers. It takes a group of 4 roofers 9 hours to roof the house. How long does it take 6 roofers to finish the house?

- Try 359#26
- 359 #1-25 odds, 26, 31, 35, 39, 45, 47, 49

$$t = \frac{a}{n}$$
$$9 = \frac{a}{4}$$
$$a = 36$$
$$t = \frac{36}{n}$$

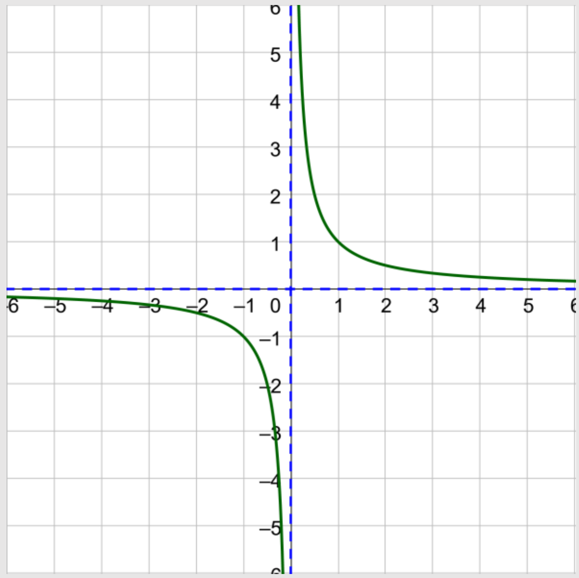
$$t = \frac{36}{6} = 6\ hour$$

$$P = \frac{k}{A}$$
$$0.43 = \frac{k}{360} \rightarrow 154.8 = k$$
$$P = \frac{154.8}{A}$$
$$P = \frac{154.8}{60} = 2.58$$


After this lesson…
• I can graph rational functions.
• I can describe transformations of rational functions.
• I can explain how to find the asymptotes of a rational function from an equation.
• I can write rational functions in different forms.
7.2 Graphing
Rational Function

7.2 Graphing Rational Functions

- Rational Functions
  - Functions written as a fraction with x in the denominator
  - $y = \frac{1}{x}$
- Shape called hyperbola
- Asymptotes
  - Horizontal: x-axis
  - Vertical: y-axis
- Domain: $x \neq 0$
- Range: $y \neq 0$



7.2 Graphing Rational Functions

- General form
  - $y = \frac{a}{x-h} + k$
    - a → stretches vertically
    - h → moves right
    - k → moves up
- How is $y = \frac{2}{x+3} + 4$ transformed from $y = \frac{1}{x}$?

Stretches vertically by factor of 2
Moves left 3
Moves up 4

7.2 Graphing Rational Functions

- How to find asymptotes
  - Vertical
    - Make the denominator = 0 and solve for *x*

7.2 Graphing Rational Functions

- Horizontal
  - Substitute a very large number for x and estimate y
- Or
  - Find the degree of numerator (N)
  - Find the degree of denominator (D)
  - If N < D, then y = 0
  - If N = D, then y = leading coefficients
  - If N > D, then no horizontal asymptote
- Find the asymptotes for $y = \frac{2x}{3x-6}$

Vertical: $3x - 6 = 0 \rightarrow 3x = 6 \rightarrow x = 2$

Horizontal: $y = \frac{2 \cdot 1000000}{3 \cdot 1000000 - 6} \rightarrow y = \frac{2}{3}$

7.2 Graphing Rational Functions

- Domain
  - All x's except for the vertical asymptotes
- Range
  - All the y's covered in the graph
  - Usually all y's except for horizontal asymptote

7.2 Graphing Rational Functions

- Graph by finding asymptotes and making a table
- Graph $y = \frac{2}{x+3} + 4$ and state the domain and range

- Try 366#9: $g(x) = \frac{4}{x} + 3$

Asymptotes
 Vertical: x + 3 = 0 → x = −3
 Horizontal: $y = \frac{2}{1000000 + 3} + 4 \rightarrow y = 4$
Domain: $x \neq -3$
Range: $y \neq 4$

Asymptotes:
 Vertical: x = 0
 Horizontal: $y = \frac{4}{1000000} + 3 \approx y = 3$
Domain: $x \neq 0$
Range: $y \neq 3$

7.2 Graphing Rational Functions

- Rewrite $g(x) = \frac{2x+5}{x+2}$ in the form $g(x) = \frac{a}{x-h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \frac{a}{x}$.

Synthetic or Long Division

$$\begin{array}{r|rr} -2 & 2 & 5 \\ & & -4 \\ \hline & -2 & 1 \end{array}$$

$$g(x) = -2 + \frac{1}{x+2} = \frac{1}{x+2} - 2$$

Domain: $x \neq -2$
Range: $y \neq -2$

7.2 Graphing Rational Functions

- Try 366#31
- Rewrite $g(x) = \frac{5x+6}{x+1}$ in the form $g(x) = \frac{a}{x-h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \frac{a}{x}$.

- 366 #~~1~~, 5, 9, 13, 17, ~~19~~, 21, 25, 29, 31, ~~35~~, 39, 41, ~~43, 45~~, 57, 59, 61, 63, 67

Synthetic or Long Division

$$\begin{array}{r|rr} -1 & 5 & 6 \\ & & -5 \\ \hline & 5 & \underline{1} \end{array}$$

$$g(x) = 5 + \frac{1}{x+1} = \frac{1}{x+1} + 5$$

Domain: $x \neq -1$
Range: $y \neq 5$

7.3 Multiplying and Dividing Rational Expressions

After this lesson…

- I can simplify rational expressions and identify any excluded values.
- I can multiply rational expressions.
- I can divide rational expressions.

7.3 Multiplying and Dividing Rational Expressions

- Simplified form
  - Numerator and denominator can have no common factors
- Steps to simplify
  - Factor numerator and denominator
  - Cancel any common factors

7.3 Multiplying and Dividing Rational Expressions

Simplify

374#5

- $\frac{x^2+11x+18}{x^3+8}$

- Try 374#1
- $\frac{2x^2}{3x^2-4x}$

$$\frac{(x+9)(x+2)}{(x+2)(x^2-2x+4)} \rightarrow \frac{x+9}{x^2-2x+4}$$

$$\frac{2xx}{x(3x-4)} \rightarrow \frac{2x}{3x-4}$$

7.3 Multiplying and Dividing Rational Expressions

- Multiplying Rational Expressions
  - Factor numerators and denominators
  - Multiply across top and bottom
  - Cancel factors

7.3 Multiplying and Dividing Rational Expressions

- 374#15
- $\frac{x^2+3x-4}{x^2+4x+4} \cdot \frac{2x^2+4x}{x^2-4x+3}$

- Try 374#13
- $\frac{x^2-3x}{x-2} \cdot \frac{x^2+x-6}{x}$

$$\frac{(x+4)(x-1)}{(x+2)(x+2)} \cdot \frac{2x(x+2)}{(x-3)(x-1)} \rightarrow \frac{2x(x+4)(x-1)(x+2)}{(x+2)(x+2)(x-3)(x-1)} \rightarrow \frac{2x(x+4)}{(x+2)(x-3)}$$

$$\frac{x(x-3)}{(x-2)} \cdot \frac{(x+3)(x-2)}{x} \rightarrow (x-3)(x+3)$$

7.3 Multiplying and Dividing Rational Expressions

- Dividing Rational Expressions
  - Take reciprocal of divisor
  - Multiply
- 374#27
- $\frac{x^2-x-6}{x+4} \div (x^2-6x+9)$
- Try 374#25
- $\frac{x^2-x-6}{2x^4-6x^3} \div \frac{x+2}{4x^3}$

$$\frac{x^2-x-6}{x+4} \cdot \frac{1}{x^2-6x+9} \rightarrow \frac{(x-3)(x+2)}{(x+4)} \cdot \frac{1}{(x-3)(x-3)} \rightarrow \frac{x+2}{(x+4)(x-3)}$$

$$\frac{x^2-x-6}{2x^4-6x^3} \cdot \frac{4x^3}{x+2} \rightarrow \frac{(x-3)(x+2)}{2x^3(x-3)} \cdot \frac{4x^3}{x+2} \rightarrow 2$$

7.3 Multiplying and Dividing Rational Expressions

- Combined Operations
  - Do the first two operations
  - Use that result with the next operation

- 374 #1, 5, 7, 9, 11, 13, 15, 17, 19, 23, 25, 27, 29, 31, 33, 43, 45, 47, 49, 55

7.4 Adding and Subtracting Rational Expressions

After this lesson…

- I can add and subtract rational expressions with like denominators.
- I can explain how to find a common denominator for rational expressions.
- I can add and subtract rational expressions with unlike denominators.

7.4 Adding and Subtracting Rational Expressions

- **Work with a partner.**
- **a.** Explain how to find each sum or difference.
- **i.** $\frac{3}{8}+\frac{1}{8}$
- **ii.** $\frac{9}{10}-\frac{3}{10}$
- **iii.** $\frac{1}{2}+\frac{3}{4}$
- **iv.** $\frac{5}{8}-\frac{7}{12}$

7.4 Adding and Subtracting Rational Expressions

- Adding and Subtracting
  - Need least common denominator (LCD)
    - If LCD already present, add or subtract numerators only

  - To get fractions with LCD
    - Factor all denominators
    - LCD is the product of the highest power of each factor in either expression
    - Whatever you multiply the denominator by, multiply the numerator also

7.4 Adding and Subtracting Rational Expressions

- 382#9
- Find the least common multiple of $5x$ and $5x - 10$.

- Try 382#11
- $2x^2 - 18, x^2 + x - 12$

$$5x = 5 \cdot x$$
$$5x - 10 = 5 \cdot (x - 2)$$

LCM: $5 \cdot x \cdot (x - 2) = 5x(x - 2)$

$$2x^2 - 18 = 2(x^2 - 9) = 2(x - 3)(x + 3)$$
$$x^2 + x - 12 = (x + 4)(x - 3)$$

LCM: $2(x - 3)(x + 3)(x + 4)$

7.4 Adding and Subtracting Rational Expressions

- 382#1
- $\frac{15}{4x}+\frac{5}{4x}$

- Try 382#5
- $\frac{5x}{x+3}+\frac{15}{x+3}$

$$\frac{20}{4x} \rightarrow \frac{5}{x}$$

$$\frac{5x+15}{x+3} \rightarrow \frac{5(x+3)}{x+3} \rightarrow 5$$

7.4 Adding and Subtracting Rational Expressions

- 382#19
- $\frac{12}{x^2+5x-24}+\frac{3}{x-3}$

- Try 382#17
- $\frac{3}{x+4}-\frac{1}{x+6}$

$$\frac{12}{(x-3)(x+8)}+\frac{3}{x-3}\to\frac{12}{(x-3)(x+8)}+\frac{3(x+8)}{(x-3)(x+8)}$$
$$\to\frac{12}{(x-3)(x+8)}+\frac{3x+24}{(x-3)(x+8)}\to\frac{3x+36}{(x-3)(x+8)}\to\frac{3(x+12)}{(x-3)(x+8)}$$

$$\frac{3}{x+4}-\frac{1}{x+6}\to\frac{3(x+6)}{(x+4)(x+6)}-\frac{1(x+4)}{(x+4)(x+6)}\to\frac{3x+18}{(x+4)(x+6)}-\frac{x+4}{(x+4)(x+6)}$$
$$\to\frac{2x+14}{(x+4)(x+6)}\to\frac{2(x+7)}{(x+4)(x+6)}$$

7.4 Adding and Subtracting Rational Expressions

- Simplifying Complex Fractions
  - Fractions within fractions

  - Follow order of operations (groups first)
  - Divide

7.4 Adding and Subtracting Rational Expressions

- 382#39
- $\dfrac{\frac{1}{3x^2-3}}{\frac{5}{x+1}-\frac{x+4}{x^2-3x-4}}$

- Try 382#35
- $\dfrac{\frac{x}{3}-6}{10+\frac{4}{x}}$

- 382 #1, 5, 7, 9, 11, 13, 15, 17, 19, 20, 21, 23, 35, 39, 41, 55, 57, 59, 61, 65

$$\frac{\frac{1}{3x^2-3}}{\frac{5}{x+1}-\frac{x+4}{x^2-3x-4}} \rightarrow \frac{\frac{1}{3(x-1)(x+1)}}{\frac{5}{x+1}-\frac{x+4}{(x-4)(x+1)}} \rightarrow \frac{\frac{1}{3(x-1)(x+1)}}{\frac{5(x-4)}{(x-4)(x+1)}-\frac{x+4}{(x-4)(x+1)}}$$

$$\rightarrow \frac{\frac{1}{3(x-1)(x+1)}}{\frac{5x-20}{(x-4)(x+1)}-\frac{x+4}{(x-4)(x+1)}} \rightarrow \frac{\frac{1}{3(x-1)(x+1)}}{\frac{4x-24}{(x-4)(x+1)}} \rightarrow \frac{\frac{1}{3(x-1)(x+1)}}{\frac{4(x-6)}{(x-4)(x+1)}}$$

$$\rightarrow \frac{1}{3(x-1)(x+1)} \cdot \frac{(x-4)(x+1)}{4(x-6)} \rightarrow \frac{x-4}{12(x-1)(x-6)}$$

$$\frac{\frac{x}{3}-6}{10+\frac{4}{x}} \rightarrow \frac{\frac{x}{3}-\frac{18}{3}}{\frac{10x}{x}+\frac{4}{x}} \rightarrow \frac{\frac{x-18}{3}}{\frac{10x+4}{x}} \rightarrow \frac{\frac{x-18}{3}}{\frac{2(5x+2)}{x}} \rightarrow \frac{x-18}{3} \cdot \frac{x}{2(5x+2)} \rightarrow \frac{x(x-18)}{6(5x+2)}$$

7.5 Solving Rational Equation

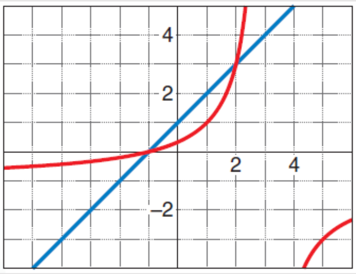
After this lesson…

- I can solve rational equations by cross multiplying and by using least common denominators.
- I can identify extraneous solutions of rational equations.
- I can solve real-life problems using inverses of rational functions.

7.5 Solving Rational Equations

- **Work with a partner.**
- **a.** Match each equation with its related graph. Explain your reasoning. Then use the graph to approximate the solution(s) of the equation.
- **i.** $\frac{2}{x-1} = 1$ **ii.** $\frac{-x-1}{x-3} = x + 1$ **iii.** $\frac{1}{x} = \frac{-1}{x-2}$

A.



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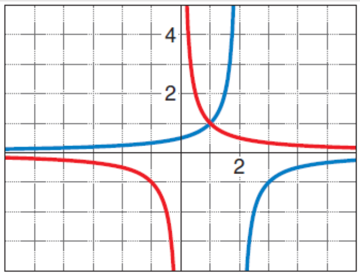

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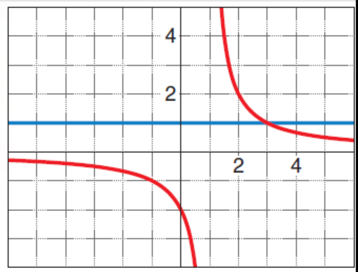


7.5 Solving Rational Equations

- Only when the = sign is present!!!

- Method 1: simplify both sides and cross multiply

- Method 2:
- Multiply both sides by LCD to remove fractions
- Solve
- Check answers

7.5 Solving Rational Equations

- 390#5
- $\frac{x}{2x+7} = \frac{x-5}{x-1}$

- 390#1
- $\frac{4}{2x} = \frac{5}{x+6}$

Cross multiply

$$x(x-1) = (2x+7)(x-5) \rightarrow x^2 - x = 2x^2 - 3x - 35 \rightarrow 0 = x^2 - 2x - 35 \rightarrow 0 = (x-7)(x+5) \rightarrow x = 7, -5$$

$$10x = 4(x+6) \rightarrow 10x = 4x + 24 \rightarrow 6x = 24 \rightarrow x = 4$$

7.5 Solving Rational Equations

- 390#21
- $\frac{6x}{x+4} + 4 = \frac{2x+2}{x-1}$

- Try 390#17
- $\frac{3}{2} + \frac{1}{x} = 2$

Cross multiply (simplify each side of = first)

$$\frac{6x}{x+4} + \frac{4(x+4)}{x+4} = \frac{2x+2}{x-1} \to \frac{6x}{x+4} + \frac{4x+16}{x+4} = \frac{2x+2}{x-1} \to \frac{10x+16}{x+4} = \frac{2x+2}{x-1}$$
$$\to (10x+16)(x-1) = (2x+2)(x+4) \to 10x^2 + 6x - 16 = 2x^2 + 10x + 8$$
$$\to 8x^2 - 4x - 24 = 0 \to 4(2x^2 - x - 6) = 0 \to 4(2x+3)(x-2) = 0 \to x = -\frac{3}{2}, 2$$

OR multiply by LCD

$$\frac{6x(x-1)(x+4)}{x+4} + 4(x-1)(x+4) = \frac{(2x+2)(x-1)(x+4)}{x-1}$$
$$\to 6x(x-1) + 4(x-1)(x+4) = (2x+2)(x+4) \to 6x^2 - 6x + 4x^2 + 12x - 16$$
$$= 2x^2 + 10x + 88x^2 - 4x - 24 = 0 \to 4(2x^2 - x - 6) = 0 \to 4(2x+3)(x-2) = 0$$
$$\to x = -\frac{3}{2}, 2$$

Cross multiply (simplify each side of = first)

$$\frac{3x}{2x} + \frac{1(2)}{x(2)} = 2 \to \frac{3x+2}{2x} = \frac{2}{1} \to 3x + 2 = 2(2x) \to 3x + 2 = 4x \to 2 = x$$

OR multiply by LCD

$$\frac{3}{2} + \frac{1}{x} = 2 \to \frac{3(2x)}{2} + \frac{1(2x)}{x} = 2(2x) \to 3x + 2 = 4x \to 2 = x$$

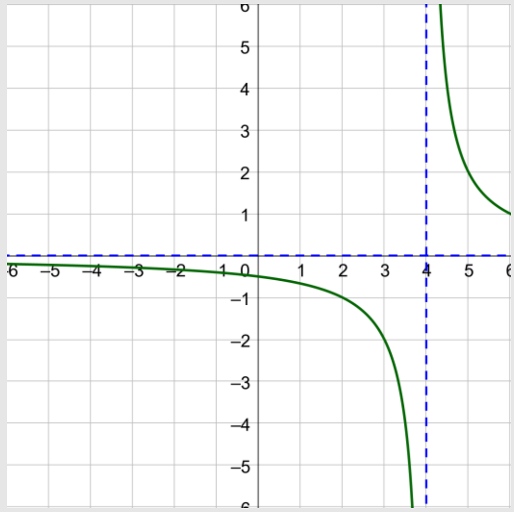
7.5 Solving Rational Equations

- Determine if the inverse of a function is a function
  - Graph the function
  - If any horizontal line touches the graph more than once, then the inverse is not a function

- Finding Inverse of Rational Functions
  - Switch x and y
  - Solve for y

7.5 Solving Rational Equations

- 391#35 $f(x) = \frac{2}{x-4}$. Determine whether the inverse of f is a function. Then find the inverse.



The graph passes the horizontal line test, so the inverse is a function

$$y = \frac{2}{x-4}$$

Switch x and y

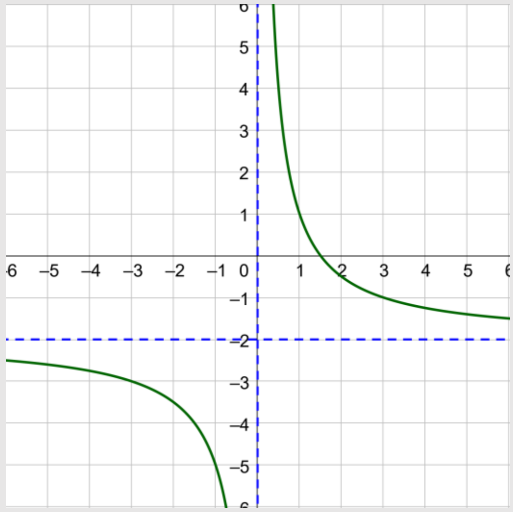
$$x = \frac{2}{y-4}$$

Solve for y

$$(y-4)x = 2$$
$$y - 4 = \frac{2}{x}$$
$$y = \frac{2}{x} + 4$$

7.5 Solving Rational Equations

- Try 391#37 $f(x) = \frac{3}{x} - 2$. Determine whether the inverse of f is a function. Then find the inverse.



- 390 #1, 5, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 35, 37, 39, 57, 59, 61, 63, 65

The graph passes the horizontal line test, so the inverse is a function

$$y = \frac{3}{x} - 2$$

Switch x and y

$$x = \frac{3}{y} - 2$$

Solve for y

$$x + 2 = \frac{3}{y}$$

$$y(x + 2) = 3$$

$$y = \frac{3}{x + 2}$$